Volume Rendering

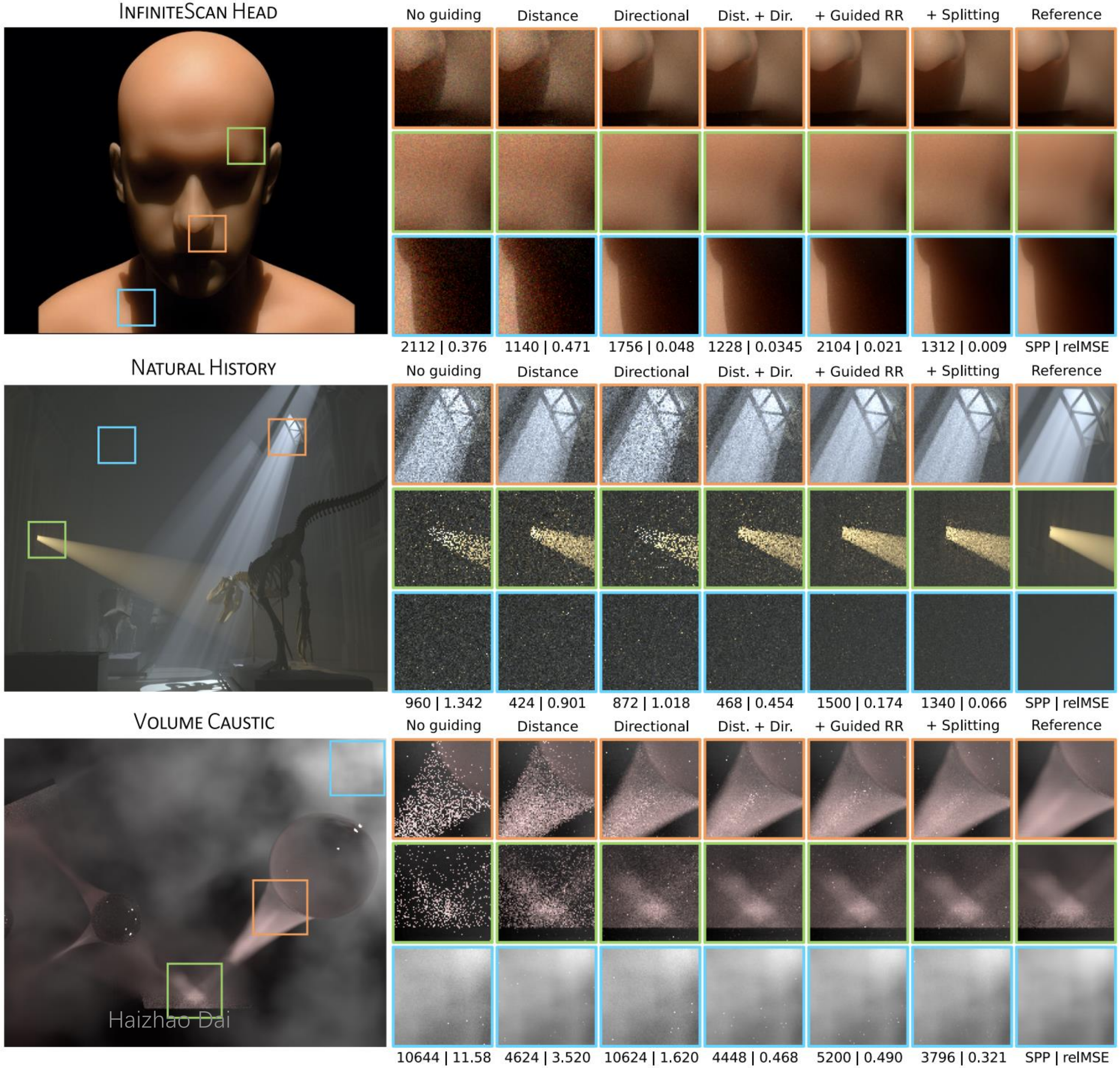
A Physically Based Rendering Technique Used in NeRF.

Haizhao Dai 2022/08/19

Gallery

Spectral and Decomposition Tracking for Rendering Heterogeneous Volumes (Siggraph 2017)

Volume Path Guiding Based on Zero-Variance Random Walk Theory (Siggraph 2019)



What Is Rendering?

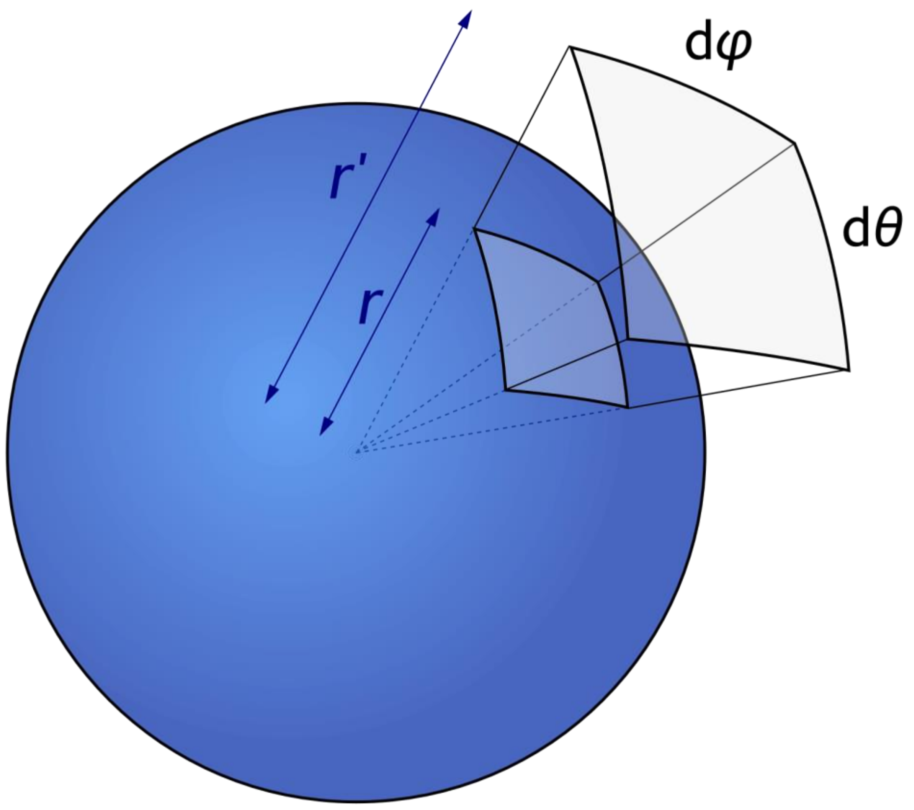
- In computer graphics, rendering or image synthesis is the process of generating a photorealistic or non-photorealistic image from a model by means of a computer program.
- Photorealistic & Stylized
- Physically based rendering (PBR).

Models of Light

- Quantum optics: wave-particle duality.
- Wave optics: electromagnetic wave.
  - A Generic Framework for Physical Light Transport (Siggraph 2021)
- Geometric optics: straight line.
  - Ray equation: $\boldsymbol{r}(t) = \boldsymbol{o} + t\boldsymbol{d}$ / $\boldsymbol{r}(t) = \boldsymbol{o} + t\boldsymbol{\Theta}$
  - Geodesic: Gravitational Lensing by Spinning Black Holes in Astrophysics, and in the Movie Interstellar (ArXiv 1502.03808)

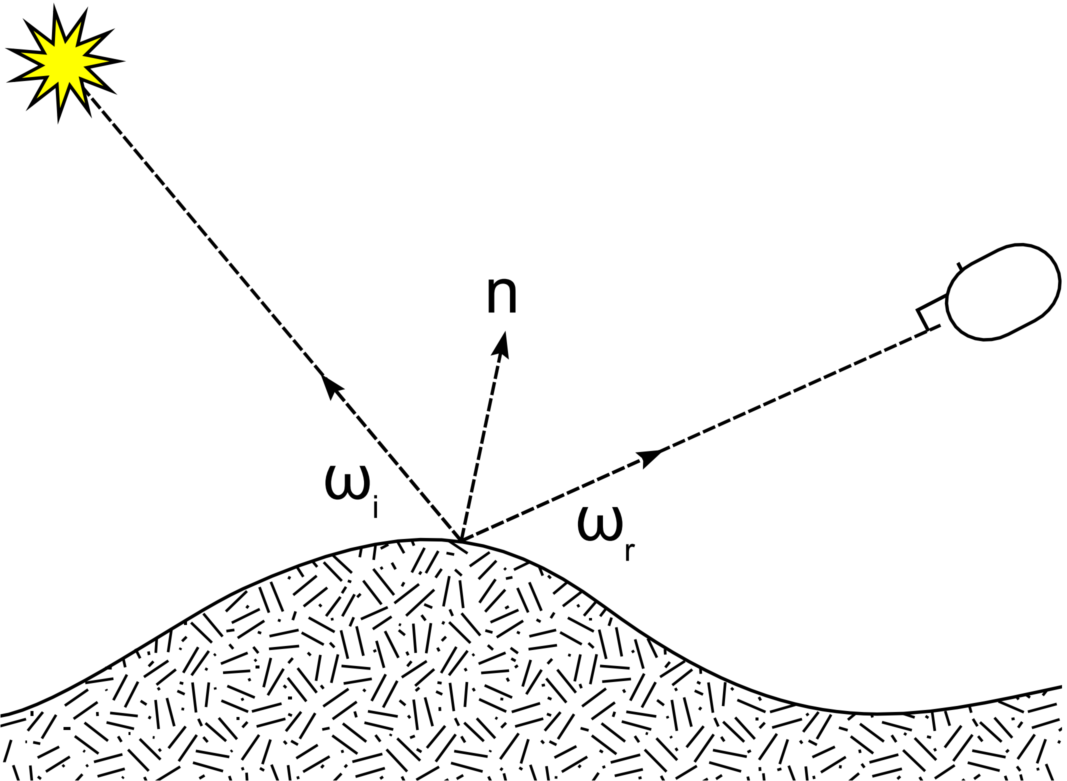
Radiometry

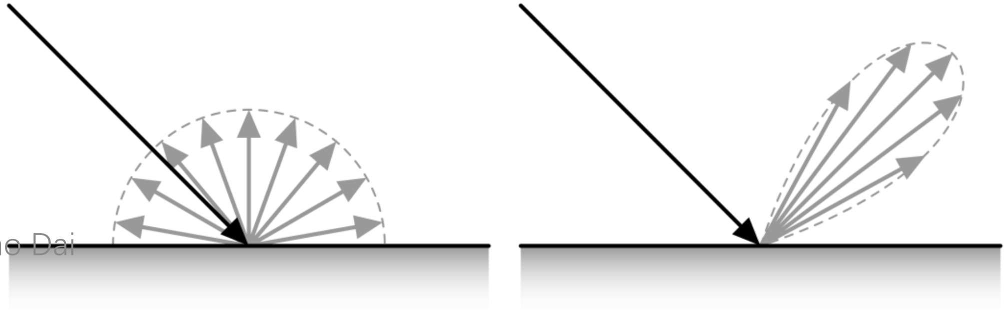
- Flux
  - The total energy flows through a surface per unit time, symbol Φ, unit $[W]$.
- Irradiance/Radiosity
  - The incident/exitant radiant power on a surface, per unit surface area, symbol E/B, unit $[W/m^2]$.
  - $E = B = \frac{\mathrm{d}\Phi}{\mathrm{d}A}$
- Radiance
  - All the direction of power arrives at or leaves from a certain point on surface, per unit solid angle, per unit projected area, symbol L, unit $[W/(m^2 \cdot \mathrm{sr})]$.
  - $L = \frac{\mathrm{d}^2\Phi}{\mathrm{d}A^{\perp}\mathrm{d}\omega}$



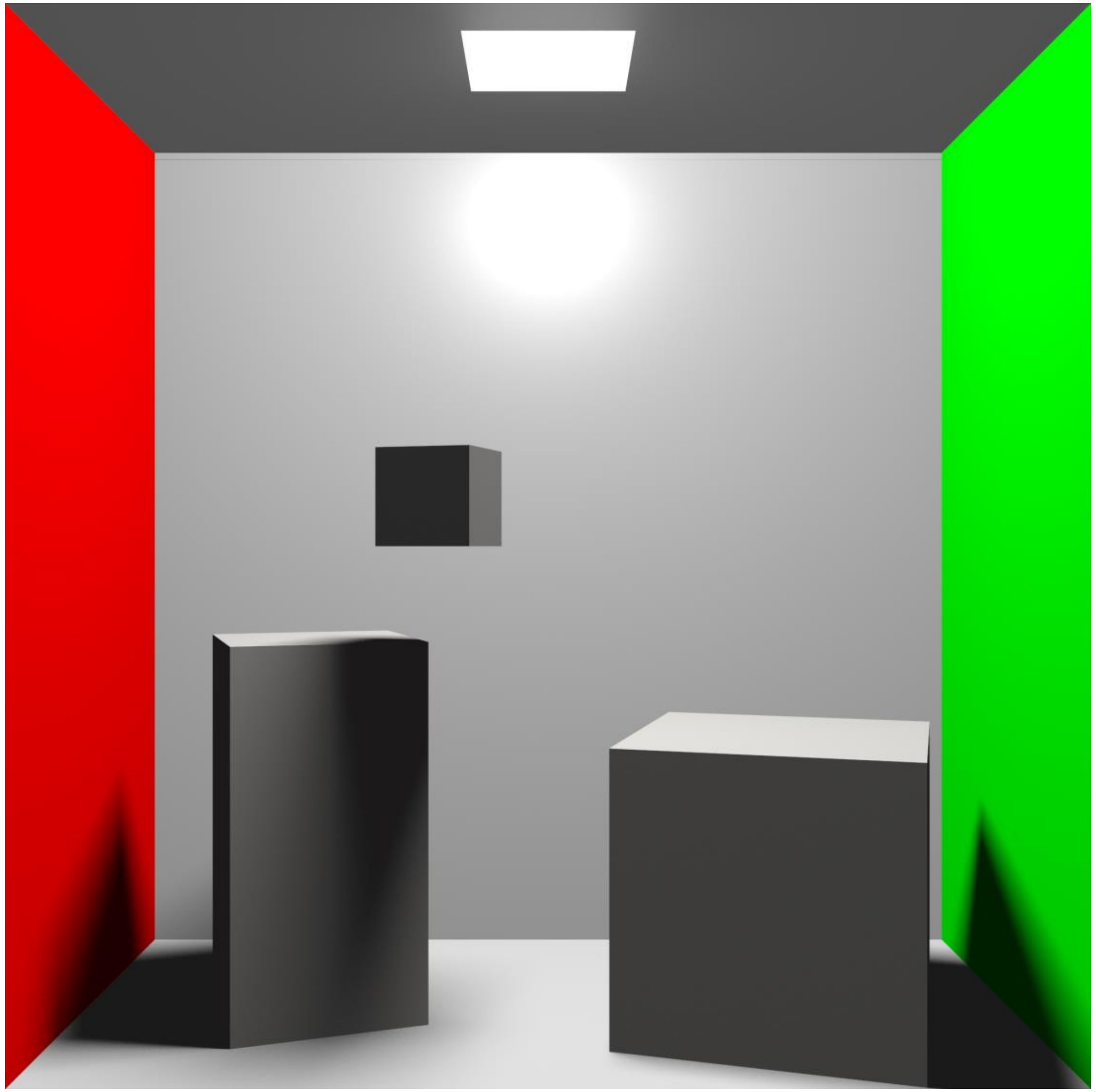
Surface Rendering

- Bidirectional Reflectance Distribution Function (BRDF)
- Rendering equation
  - $L(\boldsymbol{x}, \boldsymbol{\Theta}) = L_e(\boldsymbol{x}, \boldsymbol{\Theta}) + \int_{\mathcal{H}^2} L(\boldsymbol{x}, \boldsymbol{\Psi}) f(\boldsymbol{x}, \boldsymbol{\Theta} \leftrightarrow \boldsymbol{\Psi}) |\cos(\boldsymbol{N}_{\boldsymbol{x}}, \boldsymbol{\Psi})| \mathrm{d}\omega_{\boldsymbol{\Psi}}$
- Global illumination/Path tracing





Results

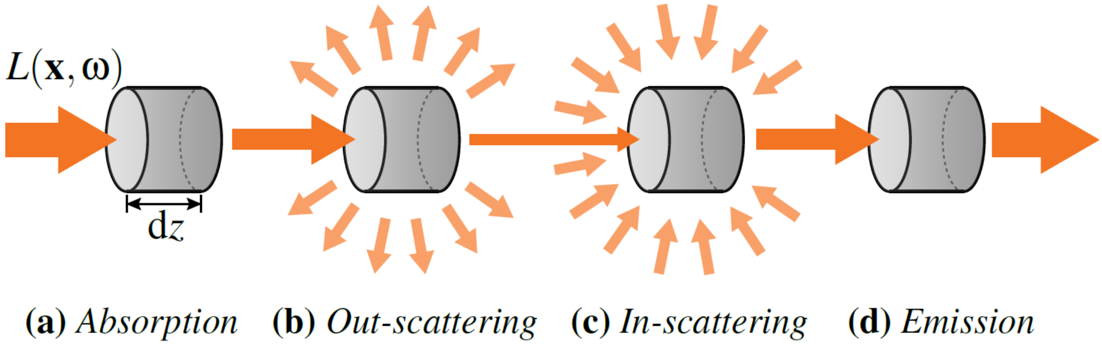


Direct illumination

Path tracing (16 times)

Participating Media

- Oversimplification
  - No participating media. Radiance is conserved along its path.
  - Ray scatters from the same location where it hits an objects.
  - Can not clearly define a surface for smoke, cloud, and fire.
- Participating media
  - Transform into other forms of energy. (Absorption)
  - Transform into visible light photos. (Emission)
  - Change the transport direction. (Scattering)



- Volumetric representation
- Suppose there is a ray transferring from $\boldsymbol{x}$ to $\boldsymbol{y}$, with ray equation:
  - $\boldsymbol{r}(t) = \boldsymbol{x} + \boldsymbol{\Theta} t,\ \boldsymbol{\Theta} = \frac{\boldsymbol{y}-\boldsymbol{x}}{||\boldsymbol{y}-\boldsymbol{x}||},\ 0 \le t \le s = ||\boldsymbol{y} - \boldsymbol{x}||$
  - Note that the ray start from the light source.

Absorption

- Absorption coefficient $\sigma_a(\boldsymbol{z})$ (unit $[m^{-1}]$).
- The probability of a photon get absorbed in a volume.
- In general, absorption is isotropic (w.r.t. input direction).

$$\mathrm{d}L(\mathbf{z} \longrightarrow \Theta) = -\sigma_a(\mathbf{z})L(\mathbf{z} \longrightarrow \Theta)\mathrm{d}t$$

Emission

- Volume emittance: $\epsilon(\boldsymbol{z})$ (units $[W/m^3]$)
- In general, emission is isotropic (w.r.t. output direction): $\epsilon(\boldsymbol{z})/4\pi$ (units $[W/(m^3 \cdot \mathrm{sr})]$)
- The radiance added to the ray due to the volume emittance:

$$\mathrm{d}L(\mathbf{z} \rightarrow \Theta) = \frac{\epsilon(\mathbf{z})}{4\pi}\mathrm{d}t = L_e(\mathbf{z})\mathrm{d}t$$

Out-scattering

- Scattering coefficient $\sigma_s(\boldsymbol{z})$ (unit $[m^{-1}]$).
- Out-scattering is identical to absorption.
- Extinction coefficient $\sigma_t(\boldsymbol{z}) = \sigma_a(\boldsymbol{z}) + \sigma_s(\boldsymbol{z})$ (unit $[m^{-1}]$).

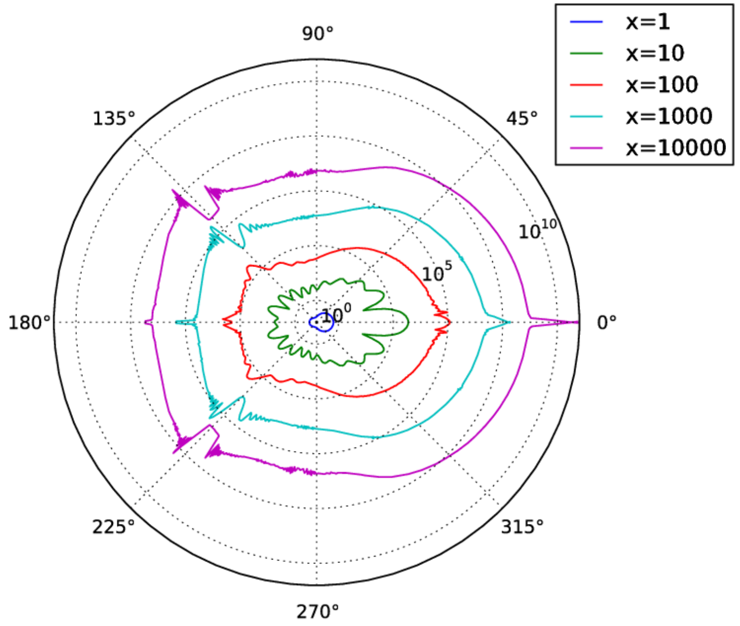
$$\mathrm{d}L(\mathbf{z} \longrightarrow \Theta) = -\sigma_s(\mathbf{z})L(\mathbf{z} \longrightarrow \Theta)\mathrm{d}t$$

In-scattering

- Scattering is not isotropic (w.r.t. output direction).
- Phase function.

$$\mathrm{d}L(\mathbf{z} \longrightarrow \Theta) = \sigma_s(\mathbf{z}) \int_{S^2} p(\mathbf{z}, \Theta \leftrightarrow \Psi) L(\mathbf{z} \longrightarrow \Psi) \mathrm{d}\omega_\Psi \mathrm{d}t$$

$$= \sigma_s(\mathbf{z}) L_s(\mathbf{z} \longrightarrow \Theta) \mathrm{d}t$$



Radiative Transfer Equation

- A kind of Boltzmann equation.
- $(\Theta \cdot \nabla)L(\mathbf{z} \longrightarrow \Theta) = -\sigma_t(\mathbf{z})L(\mathbf{z} \longrightarrow \Theta) + \sigma_a(\mathbf{z})L_e(\mathbf{z} \longrightarrow \Theta) + \sigma_s(\mathbf{z})L_s(\mathbf{z} \longrightarrow \Theta)$
- Solution to first-order homogeneous linear ODE:

$$\frac{\mathrm{d}}{\mathrm{d}t}L(t) = -\sigma_t(t)L(t)$$

$$\frac{\mathrm{d}L(t)}{L(t)} = -\sigma_t(t)\mathrm{d}t$$

$$\ln L(t) - \ln L(0) = -\int_0^t \sigma_t(u)\mathrm{d}u$$

$$L(t) = L(0)\exp\left(-\int_0^t \sigma_t(u)\mathrm{d}u\right)$$

Transmittance

- Optical thickness: $\tau(\boldsymbol{x}, \boldsymbol{z}) / \tau(0, t)$.
  - $\tau(0, t) = -\int_0^t \sigma_t(u) \mathrm{d}u$
- Transmittance: $T(\boldsymbol{x}, \boldsymbol{z}) / T(0, t)$.
  - Surface definition: The radiant flux transmitted by that surface / The radiant flux received by that surface.
  - $L(t)/L(0) = T(0, t)$
  - The percentage of radiance reserved (Statistically)
  - The probability of a photo pass through (Probability)

Radiative Transfer Equation

- Solution to first-order inhomogeneous linear ODE:

$$L(t) = M(t)\exp\left(-\int_0^t \sigma_t(u)\mathrm{d}u\right)$$

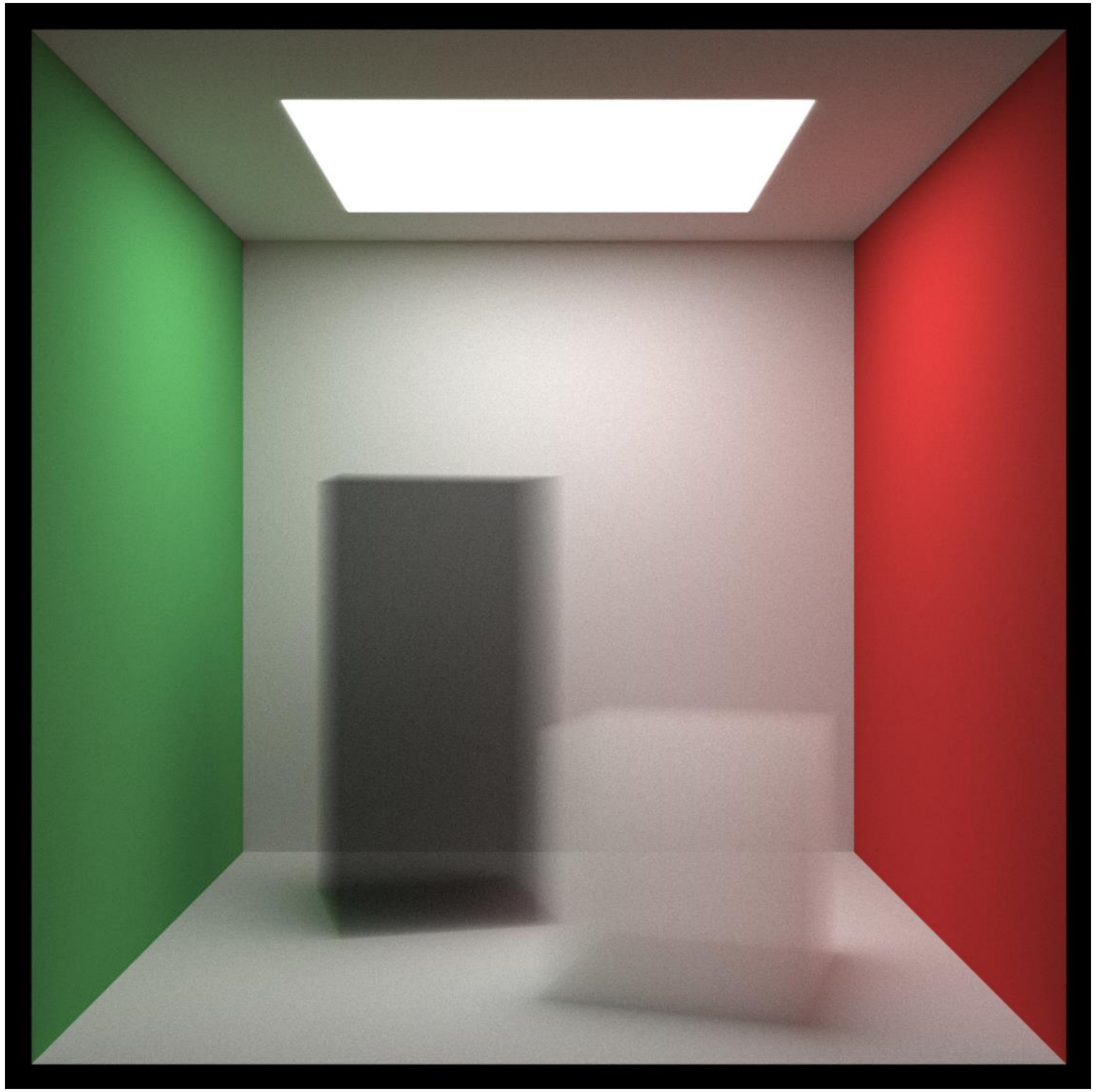
$$\mathrm{d}M(t) = \exp\left(\int_0^t \sigma_t(u)\mathrm{d}u\right)[\sigma_a(t)L_e(t) + \sigma_s(t)L_s(t)]\mathrm{d}t$$

$$M(t) - M(0) = \int_0^t \exp\left(\int_0^u \sigma_t(v)\mathrm{d}v\right)[\sigma_a(u)L_e(u) + \sigma_s(u)L_s(u)]\mathrm{d}u$$

$$L(t) = L(0)\exp\left(-\int_0^t \sigma_t(u)\mathrm{d}u\right) + \int_0^t \exp\left(-\int_u^t \sigma_t(v)\mathrm{d}v\right)[\sigma_a(u)L_e(u) + \sigma_s(u)L_s(u)]\mathrm{d}u$$

$$L(t) = L(0)T(0,t) + \int_0^t T(u,t)[\sigma_a(u)L_e(u) + \sigma_s(u)L_s(u)]\mathrm{d}u$$

Results



Path tracing (50 times) + volume rendering

NeRF's Setting

- No scattering.
- Note that the ray is from camera and reversely parameterized.
- Discretize:
  - Non-uniformly divide $[0, s]$ into n bins, x_i $(0 \leq i \leq n)$ is the points, $\delta_i = x_i - x_{i-1}$ $(1 \leq i \leq n)$ is the length of the small bin, using Riemann sum to approximate the integral.

$$L(s) = L(0)T(0,s) + \int_0^s T(t,s)\sigma_t(t)L_e(t)\mathrm{d}t$$

$$T(t,s) = \exp\left(-\int_t^s \sigma_t(u)\mathrm{d}u\right)$$

Discretizing

- Non-uniformly divide $[0, s]$ into n parts, x_i is the points, δ_i is the length of the small bin, using Riemann sum to approximate the integral.
- Transmittance:

$$\begin{aligned} T(0, s) &= \exp\left(-\int_0^s \sigma_t(u)\mathrm{d}u\right) \\ &= \exp\left(-\sum_{i=1}^{n} \sigma_t(x_i)\delta_i\right) \\ &= \prod_{i=1}^{n} \exp(-\sigma_t(x_i)\delta_i) \end{aligned}$$

Discretizing

- Non-uniformly divide $[0, s]$ into n parts, x_i is the points, δ_i is the length of the small bin, using Riemann sum to approximate the integral.
- Radiance:

$$
\begin{aligned}
\int_0^s T(t,s)\sigma_t(t)L_e(t)\mathrm{d}t &= \sum_{i=1}^n \int_{x_{i-1}}^{x_i} T(t,s)\sigma_t(t)L_e(t)\mathrm{d}t \\
&\approx \sum_{i=1}^n \int_{x_{i-1}}^{x_i} T(t,s)\sigma_t(t)L_e(x_i)\mathrm{d}t = \sum_{i=1}^n L_e(x_i) \int_{x_{i-1}}^{x_i} T(t,s)\sigma_t(t)\mathrm{d}t \\
&= \sum_{i=1}^n L_e(x_i) \int_{x_{i-1}}^{x_i} \frac{\mathrm{d}}{\mathrm{d}t} \exp\left(-\int_t^s \sigma_t(u)\mathrm{d}u\right) \mathrm{d}t = \sum_{i=1}^n L_e(x_i)\left[T(x_i,s) - T(x_{i-1},s)\right] \\
&= \sum_{i=1}^n L_e(x_i)T(x_i,s)\left[1 - T(x_{i-1},x_i)\right] \approx \sum_{i=1}^n L_e(x_i)T(x_i,s)\left[1 - \exp\left(-\int_{x_{i-1}}^{x_i} \sigma_t(x_i)\mathrm{d}u\right)\right] \\
&= \sum_{i=1}^n L_e(x_i)T(x_i,s)\left[1 - \exp(-\sigma_t(x_i)\delta_i)\right]
\end{aligned}
$$

Thus we define $\alpha_i = 1 - \exp(\sigma_t(x_i)\delta_i)$

References

- Advanced Global Illumination. Dutré et al. 2006.
- Spectral and Decomposition Tracking for Rendering Heterogeneous Volumes. Kutz et al. Siggraph 2017.
- Monte Carlo Methods for Volumetric Light Transport Simulation. Novák et al. Eurographics 2018.
- Volume Path Guiding Based on Zero-Variance Random Walk Theory. Herholz et al. Siggraph 2019.
- Ray Tracing in One Weekend Series. Peter Shirley. https://raytracing.github.io 2020.